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THE SECOND LIAPUNOV METHOD IN THE THEORY OF PHASE SYNCHRONIZATION

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A certain analog of the Liapunov second method is constructed for dynamic systems with cylindrical phase space. The known results obtained by it for second order dynamic systems are extended to systems with cylindrical phase space of arbitrary dimensions. The derived theorems are used for analyzing the operation of a system of two synchronous machines and for investigating the automatic phase frequency as a "whole".

The working modes of systems of automatic phase frequency control (APFC) are usually such that the phase difference $\sigma(t)$ between the reference generator that is being synchronized is a bounded function of time $t \in (0, +\infty)$. It is often possible to establish on the basis of boundedness of $\sigma(t)$ that for $t \to +\infty$ there exists a finite limit of $\sigma(t)$ for autononmous APFC systems [1, 2]. The presence of such limit means that the considered working mode of the APFC is one of capture [1]. Similar statements are also valid for working modes of synchronous motors, except that then the phase difference between the rotating magnetic field and the rotor is represented by function $\sigma(t)$ [3—6].

A certain analog of the Liapunov second method is derived below, which makes it possible to obtain effective sufficient conditions of boundedness or unboundedness for function

$\sigma(t)$. The problem of boundedness of function $\sigma(t)$ is reduced by that method to a simultaneous construction of some function of the Liapunov kind and to resolving the question of boundedness of all solutions of some second order differential equations of the form

$$\theta^{\cdot\cdot} + R(\theta)\,\theta^{\cdot} + f(\theta) = 0 \qquad (0.1)$$

where $R(\theta)$ and $f(\theta)$ are some 2π-periodic functions. We note in connection with this that Eq. (0.1) has been thoroughly investigated for many important classes of functions $R(\theta)$ and $f(\theta)$, particularly with respect to the boundedness of solutions of Eq. (0.1) for $R(\theta) \equiv$ const. Numerous results of investigations of Eq. (0.1) and a list of works dealing with this subject are given in the monographs [1, 7].

1. We introduce in the analysis continuous functions $f(\sigma)$, $u(\sigma)$ and $v(\sigma)$ that are specified for $\sigma \in (-\infty, +\infty)$, continuous functions $W(t)$ and $\psi(t)$ determined for $t \geqslant 0$, and the continuously differentiable function $\sigma(t)$. In what follows we assume that the inequalities $u(\sigma) > 0$ and $v(\sigma) > 0$ are satisfied for all σ.

We further assume that function $f(\sigma)$ satisfies condition A, if it is continuously differentiable, 2π-periodic, has zeros, and if the inequality $f'(\sigma) \neq 0$ holds for any σ that satisfies the condition $f(\sigma) = 0$.

Theorem 1. Let the 2π-periodic functions $u(\sigma)$ and $v(\sigma)$ be continuously differentiable, function $f(\sigma)$ satisfies condition A, and let the following requirements be also satisfied:

1) all solutions $\theta(t)$ of the second order differential equation

$$\theta^{\cdot\cdot} + 2\sqrt{u(\theta)\,v(\theta)}\,\theta^{\cdot} + f(\theta) = 0$$

are bounded in the interval $(0, +\infty)$;

2) the inequality $W(t) > 0$ is satisfied for all $t \geqslant 0$ for which $f(\sigma(t)) = 0$ and $f'(\sigma(t)) < 0$, and

3) function

$$W(t) + \int_0^t [2u(\sigma(\tau))\,W(\tau) + v(\sigma(\tau))(\sigma^{\cdot}(\tau))^2 + f(\sigma(\tau))\,\sigma^{\cdot}(\tau)]\,d\tau$$

is a nonincreasing function of t. Function $\sigma(t)$ is then bounded in the interval $(0, +\infty)$.

Proof. It is shown in [7] that when Condition (1) of the theorem is satisfied for any integral k, there exists a differentiable function $F_k(\sigma)$ that satisfies the following relationships:

$$F_k(\sigma)\,F_k'(\sigma) + 2\sqrt{u(\sigma)\,v(\sigma)}\,F_k(\sigma) + f(\sigma) = 0 \qquad (1.1)$$
$$\forall\sigma \in (-\infty, +\infty)$$
$$F_k(\sigma_0 - 2k\pi) = 0, \quad \lim_{\sigma\to\infty} F_k(\sigma)^2 = \infty, \quad F_k(\sigma) \equiv F_0(\sigma + 2k\pi)$$

where σ_0 is some zero of function $f(\sigma)$ in the interval $[0, 2\pi]$, and $f'(\sigma_0) < 0$. Let us now examine function

$$V_k(t) = W(t) - {}^1/_2\, F_k(\sigma(t))^2$$

From the first of equalities (1.1) we obtain

$$v(\sigma^{\cdot})^2 + uF_k^2 + F_kF_k'\sigma^{\cdot} + f\sigma^{\cdot} \geqslant uF_k^2 - [4v]^{-1}[f + F_kF_k']^2 =$$
$$[-4v]^{-1}[F_kF_k' + 2\sqrt{uv}\,F_k + f]\,[F_kF_k' - 2\sqrt{uv}\,F_k + f] = 0$$

where u, v, f and F_k are functions of $\sigma(t)$. This and Condition (3) of the theorem imply that function

$$V_k(t) + 2\int_0^t u(\sigma(\tau))\,V_k(\tau)\,d\tau \tag{1.2}$$

is also a nonincreasing function of t. But then it follows from the inequality $V_k\,(0) \leqslant 0$ that $V_k\,(t) \leqslant 0$ for all $t \geqslant 0$.

In fact, if the opposite, i.e. the existence of a number $t_1 > 0$ for which $V_k\,(t_1) > 0$, is assumed, then, owing to the continuity of function $V_k\,(t)$ and condition $V_k\,(0) \leqslant 0$, we obtain that for $t \in (t_2,\ t_1)$ there exists a number $t_2 \in [0,\ t_1)$ such that $V_k\,(t_2) = 0$ and $V_k\,(t) > 0$. But by virtue of the no-growth of function (1.2) the inequality

$$V_k\,(t_1) - V_k\,(t_2) + 2\int_{t_2}^{t_1} u\,(\sigma\,(\tau))\;V_k\,(\tau)\,d\tau \leqslant 0$$

is satisfied. Hence $V_k\,(t_1) \leqslant V_k\,(t_2) = 0$, which contradicts the assumption that $V_k(t_1) > 0$. This contradiction proves that the estimate $V_k\,(0) \leqslant 0$ follows from the inequality $V_k\,(t) \leqslant 0$ for all $t \geqslant 0$.

It follows from the last two equations in (1.1) that the inequalities

$$V_k\,(0) \leqslant 0, \quad V_{-k}\,(0) \leqslant 0, \quad |\,\sigma\,(0) - \sigma_0\,| \leqslant 2k\pi \tag{1.3}$$

are satisfied for reasonably high values of k.

Let us now take k such that inequalities (1.3) are satisfied. But then, as was shown above,

$$V_k\,(t) \leqslant 0, \quad V_{-k}\,(t) \leqslant 0, \quad \forall t \geqslant 0 \tag{1.4}$$

Let us prove that

$$|\,\sigma\,(t) - \sigma_0\,| \leqslant 2k\pi, \quad \forall t \geqslant 0 \tag{1.5}$$

Assuming the opposite, i.e. that there exists a number $t_1 > 0$ for which inequality (1.5) is not satisfied, then from the continuity of function $\sigma\,(t)$ we obtain that there exists a number $t_2 \in [0,\ t_1)$, for which $|\,\sigma\,(t_2) - \sigma_0\,| = 2k\pi$. But then $f\,(\sigma\,(t_2)) = 0$, $f'\,(\sigma\,(t_2)) < 0$, and either $F_k\,(\sigma\,(t_2)) = 0$ or $F_{-k}\,(\sigma\,(t_2)) = 0$. It follows from this and inequalities (1.4) that $W\,(t_2) \leqslant 0$. This estimate and the relationships $f(\sigma\,(t_2)) = 0$ and $f'\,(\sigma\,(t_2)) < 0$ contradict Condition (2) of the theorem. This proves the validity of (1.5), and means that function $\sigma\,(t)$ is bounded in the interval $(0, +\infty)$.

Theorem 2. Let us assume the existence of a continuously differentiable function $F\,(\sigma)$ for which the following conditions are satisfied.

1) $F\,(\sigma) > 0, \quad \forall\sigma \in (-\infty, +\infty)$ and
2) $F\,(\sigma)\,F'\,(\sigma) + \sqrt{2v\,(\sigma)}\;u\,(\sigma) F\,(\sigma) + f\,(\sigma) = 0, \quad \forall\sigma \in (-\infty, +\infty)$

Let also

3) $W\,(t) + v\,(\sigma\,(t))(\sigma^{\cdot}\,(t))^2 \geqslant 0, \quad \forall t \geqslant 0$
4) the inequality $\psi\,(t) < f\,(\sigma\,(t))$ is satisfied for all $t \geqslant 0$ for which $\sigma^{\cdot}\,(t) > 0$;
5) function

$$W\,(t) + \int_0^t [2u\,(\sigma\,(\tau))\,W\,(\tau) - \psi\,(\tau)\,\sigma^{\cdot}\,(\tau)]\,d\tau$$

is a nonincreasing function of t, and

6) the inequalities $\sigma^{\cdot}\,(0) > 0$ and $2W\,(0) < -F\,(\sigma\,(0))^{2}$ are satisfied.

Then the inequality

$$\sigma^{\cdot}\,(t) \geqslant F\,(\sigma\,(t))/\sqrt{2v\,(\sigma\,(t))} \tag{1.6}$$

is satisfied for all $t \geqslant 0$.

Proof. Let us consider function

$$V(t) = W(t) + {}^1/_2F(\sigma(t))^2$$

Condition (6) of the theorem implies that $V(0) < 0$. Let us further assume that $V(t) < 0$ for $t \in [0, T)$. Then

$$W(t) + v(\sigma^{\cdot})^2 \leqslant -{}^1/_2F^2 + v(\sigma^{\cdot})^2, \quad t \in [0, T]$$

and according to Condition (3) of the theorem $F^2 \leqslant 2v(\sigma^{\cdot})^2$. This and Condition (6) for $t \in [0, T]$ yield the estimate (1.6). But then from Condition (4) we obtain the inequality

$$\psi(t)\,\sigma^{\cdot}(t) \leqslant f(\sigma(t))\,\sigma^{\cdot}(t) - \delta(t), \quad t \in [0, T] \tag{1.7}$$

where $\delta(t)$ is some continuous positive function. Then for $t \in [0, T]$ from Condition (2) and estimate (1.6) we obtain the relationship

$$uF^2 + [f + FF']\,\sigma^{\cdot} \leqslant [\sqrt{2v}]^{-1} F\,[FF' + \sqrt{2v}\,uF + f] = 0$$

This with inequality (1.7) and Condition (5) of the theorem implies that

$$V(t) + \int_0^t [2u(\sigma(\tau))V(\tau) + \delta(\tau)]\,d\tau \tag{1.8}$$

is a nonincreasing function of t in the interval $[0, T]$.

Let us now assume that $V(T) = 0$ and select the interval (T_1, T) so small that for $t \in (T_1, T)$

$$\delta(t) > 2u(\sigma(t))\,|\,V(t)\,| \tag{1.9}$$

Since function (1.8) does not increase in $[0, T]$, the inequality

$$V(T) - V(t) + \int_t^T [2u(\sigma(\tau))V(\tau) + \delta(\tau)]\,d\tau \leqslant 0$$

is valid. This, with condition (1.9) shows that for $t \in (T_1, T)$ we have $V(T) < V(t)$. However the assumption that $V(T) = 0$ implies that $V(t) > 0$ for $t \in (T_1, T)$. This contradicts the previous assumption that for $t \in [0, T)$ we have $V(t) < 0$. The validity of inequality $V(T) < 0$ is thus proved. Hence for all $t \geqslant 0$ we have $V(t) < 0$ and, as previously shown, the estimate (1.6) holds for all $t \geqslant 0$.

2. The working of a system of two synchronous machines with stator windings of zero active resistance and a purely reactive quadruple connecting them is defined by equations of the form [6]

$$\begin{gathered}\frac{d\sigma}{dt} = \xi, \quad \frac{d\xi}{dt} = -\alpha_1\xi + \frac{\alpha_4}{\alpha_9}[\alpha_5\alpha_6 \sin\sigma_0 - \\ (\alpha_5 + i_1)(\alpha_6 + i_2)\sin(\sigma + \sigma_0)] \\ \alpha_2\frac{di_1}{dt} + \alpha_4\cos(\sigma + \sigma_0)\frac{di_2}{dt} = \alpha_4(\alpha_6 + i_2)\,\xi\sin(\sigma + \sigma_0) - \alpha_7 i_1 \\ \alpha_3\frac{di_2}{dt} + \alpha_4\cos(\sigma + \sigma_0)\frac{di_1}{dt} = \alpha_4(\alpha_5 + i_1)\,\xi\sin(\sigma + \sigma_0) - \alpha_8 i_2\end{gathered} \tag{2.1}$$

on the assumption that the damping moments of rotors of both machines are proportional to slip and to their moments of inertia. In these equations $\alpha_1, \alpha_2, \ldots, \alpha_9$ are some positive constants and the number $\sigma_0 \in [0, {}^1/_2\pi]$. We further assume that $\alpha_2\alpha_3 > \alpha_4{}^2$. Note that the last inequality is always satisfied in the case of systems with a nonbranching transmission [6].

Let us consider some nontrivial solution $\sigma(t)$, $\xi(t)$, $i_1(t)$ and $i_2(t)$ of system (2.1) by applying to it Theorem 1. For this we define functions $W(t)$ and $f(\sigma)$ as follows:

$$W(t) = {}^1/_2\alpha_9\xi(t)^2 + {}^1/_2\alpha_2 i_1(t)^2 + {}^1/_2\alpha_3 i_2(t)^2 + \alpha_4\cos(\sigma(t)+\sigma_0)\, i_1(t)\, i_2(t)$$
$$f(\sigma) = \alpha_4\alpha_5\alpha_6[\sin(\sigma+\sigma_0) - \sin\sigma_0]$$

The inequality $\alpha_2\alpha_3 > \alpha_4^2$ and the nontriviality of the considered solution directly imply that Condition (2) of the theorem is satisfied.

It will be readily seen that

$$W^{\cdot}(t) = -\alpha_7(i_1(t))^2 - \alpha_8(i_2(t))^2 - f(\sigma(t))\,\xi(t) - \alpha_1\alpha_9\xi(t)^2$$

Hence, if the numbers $\lambda > 0$ and $\varepsilon > 0$ satisfy the relationships

$$\begin{aligned}&\alpha_7 - \lambda\alpha_2 > 0, \quad \alpha_8 - \lambda\alpha_3 > 0\\ &\alpha_9(\alpha_1 - \lambda) \geqslant \varepsilon, \quad (\alpha_7 - \lambda\alpha_2)(\alpha_8 - \lambda\alpha_3) \geqslant \lambda^2\alpha_4^2\end{aligned} \tag{2.2}$$

the inequality

$$W^{\cdot}(t) + 2\lambda W(t) + f(\sigma(t))\xi(t) + \varepsilon\xi(t)^2 \leqslant 0$$

is also satisfied and, consequently, Condition (3) of Theorem 1, where $u(\sigma) \equiv \lambda$ and $v(\sigma) \equiv \varepsilon$, is valid. Thus, if positive numbers λ and ε such that inequalities (2.2) are satisfied and all solutions of the second order equation

$$\theta^{\cdot\cdot} + 2\sqrt{\varepsilon\lambda}\,\theta^{\cdot} + \alpha_4\alpha_5\alpha_6[\sin(\theta+\sigma_0) - \sin\sigma_0] = 0 \tag{2.3}$$

are bounded in the interval $(0, +\infty)$ can be found, then all conditions of Theorem 1 are satisfied, and consequently $\sigma(t)$ is bounded in the interval $(0, +\infty)$.

Setting

$$\lambda_0 = \frac{\alpha_2\alpha_8 + \alpha_3\alpha_7 - \sqrt{(\alpha_2\alpha_8 - \alpha_3\alpha_7)^2 + 4\alpha_7\alpha_8\alpha_4^2}}{2(\alpha_2\alpha_3 - \alpha_4^2)}$$

$$\varepsilon = \alpha_9(\alpha_1 - \lambda), \quad \Gamma = \begin{cases}\lambda_0(\alpha_1 - \lambda_0), & 2\lambda_0 \leqslant \alpha_1\\ 0.25\alpha_1^2, & 2\lambda_0 \geqslant \alpha_1\end{cases}$$

and using the Bohm-Hayes theorem [7 — 10], we finally obtain that when

$$\left(\sin\frac{\sigma_0}{2}\right)^2 \leqslant \frac{\alpha_9\Gamma}{\alpha_4\alpha_5\alpha_6} \tag{2.4}$$

then function $\sigma(t)$ is bounded in the interval $(0, +\infty)$.

Using the Liapunov type function derived in [6] it can be readily shown that system (2.1) is monostable [11,12] and that functions $\xi(t)$, $i_1(t)$ and $i_2(t)$ are bounded in the interval $(0, +\infty)$. Hence, if inequality (2.4) is satisfied, any solution of system (2.1) tends for $t \to +\infty$ to a certain equilibrium state and, consequently, either a dynamic or a resulting stability obtains for the considered power system under any operational conditions.

3. Dynamics of a typical autonomous system of APFC are defined by equations of the form

$$x^{\cdot} = Ax + b\varphi(\sigma), \quad \sigma^{\cdot} = c^*x + \rho\varphi(\sigma) \tag{3.1}$$

where A is the Hurwitz constant of an ($n \times n$)-matrix, b and c are constant n-vectors, ρ is a number, and $\varphi(\sigma)$ is a 2π-periodic function.

We introduce in the analysis function $\chi(p) = c^* (A - pI)^{-1} b$, where p is a complex number.

Theorem 3. Let us assume that function $\chi(p)$ is nondegenerate [13] and that there exists numbers $\varepsilon > 0$, $\lambda > 0$ and μ such that the following conditions are satisfied:

1) function $\varphi(\sigma)(1 + \mu\varphi'(\sigma))$ satisfies condition A and the inequality $1 + \mu\varphi'(\sigma) \neq 0$ holds for all $\sigma \in (-\infty, +\infty)$;

2) $\mu\lambda + \operatorname{Re}\chi(i\omega - \lambda) - \rho - \varepsilon \,|\, \chi(i\omega - \lambda) - \rho \,|^2 \geqslant 0,$
$\forall \omega \geqslant 0$

3) matrix $A + \lambda I$ is a Hurwitz matrix, and

4) all solutions $\theta(t)$ of the second order equation

$$\theta^{\cdot\cdot} + 2\sqrt{\lambda\varepsilon}\,\theta^{\cdot} + \varphi(\theta)(1 + \mu\varphi'(\theta)) = 0$$

are bounded in the interval $(0, +\infty)$.

Then any solution of system (3.1) is bounded in the interval $(0, +\infty)$.

Proof. It follows from Condition (2) of Theorem 3 by the Iakubovich-Kalman lemma [13] that there exists such constant ($n \times n$)-matrix $H = H^*$ such that for all $x \in R^n$ and $\xi \in (-\infty, +\infty)$ the inequality

$$2x^*H\,[(A + \lambda I)\,x + b\xi] - \mu\lambda\xi^2 + \xi\,(c^*x + \rho\xi) + \varepsilon\,(c^*x + \rho\xi)^2 \leqslant 0 \qquad (3.2)$$

is satisfied. For $\xi = 0$ inequality (3.2) assumes the form

$$2x^*H\,(A + \lambda I)\,x \leqslant -\varepsilon\,(c^*x)^2$$

which by Lemma 1 [14] together with the nondegeneracy of $\chi(p)$ and the Hurwitz properties of matrix $A + \lambda I$ yields the inequality $H > 0$.

We introduce in the analysis functions

$$W(t) = x(t)^*\,Hx(t) - 1/2\mu\varphi(\sigma(t))^2$$
$$f(\sigma) = \varphi(\sigma)\,(1 + \mu\varphi'(\sigma))$$

where $x(t)$ and $\sigma(t)$ are some nontrivial solutions of system (3.1). It follows immediately from the positive definiteness of matrix H, the nontriviality of the considered solution of system (3.1), and from the inequality $1 + \mu\varphi'(\sigma) \neq 0$ that Condition (2) of Theorem 1 is satisfied.

The inequality (3.2) yields the estimate

$$W^{\cdot}(t) + 2\lambda W(t) + \varepsilon\,(\sigma^{\cdot}(t))^2 + f(\sigma(t))\,\sigma^{\cdot}(t) \leqslant 0, \quad \forall t \geqslant 0$$

Thus for $u(\sigma) \equiv \lambda$ and $v(\sigma) \equiv \varepsilon$ Condition (3) of Theorem 1 is satisfied and Condition (4) of Theorem 3 coincides with Condition (1) of Theorem 1.

Since all conditions of Theorem 1 are satisfied, the solution $\sigma(t)$ of system (3.1) is bounded in the interval $(0, +\infty)$. Boundedness of solution $x(t)$ follows from the Hurwitz properties of matrix A and from the boundedness of function $\varphi(\sigma)$.

4. If the transfer function of the low-pass filter is a regular fractional-rational function, and some perturbation is present at its input, the equations of a typical system of APFC are of the form

$$x^{\cdot} = Ax + b\,[\varphi(\sigma) + g(t)], \quad \sigma^{\cdot} = c^* x \tag{4.1}$$

where A' is a constant ($n \times n$)-matrix, b and c are constant n-vectors, and $\varphi(\sigma)$ and $g(t)$ are continuous functions. Function $\varphi(\sigma)$ is 2π-periodic. We also assume that

$$g(t) \leqslant 0, \quad \forall t \geqslant 0; \quad \int_0^{2\pi} \varphi(\sigma)\, d\sigma \leqslant 0$$

Theorem 4. Let function $\chi(p) = c^*(A - pI)^{-1} b$ be nondegenerate, $c^* b < 0$, and let there exist a number $\lambda > 0$ such that the following conditions are satisfied:

1) $\operatorname{Re} \chi(i\omega - \lambda) < 0, \quad \forall \omega \geqslant 0$;

2) $\lim\limits_{\omega \to \infty} \omega^2 \operatorname{Re} \chi(i\omega - \lambda) < 0$;

3) matrix $A + \lambda I$ has one positive eigenvalue and $(n - 1)$ eigenvalues with negative real parts, and

4) the second order equation

$$\theta^{\cdot\cdot} + \sqrt{-\frac{1}{c^* b}}\, \lambda \theta^{\cdot} + \varphi(\theta) = 0$$

has a solution $\theta(t)$ that is unbounded in the interval $(0, +\infty)$. Then system (4.1) has an unbounded solution in the interval $(0, +\infty)$

Proof. It follows from Conditions (1)—(3) of Theorem 4 and the Iakubovich-Kalman lemma that there exists a number $\delta > 0$ and a constant ($n \times n$)-matrix $H = H^*$ with one negative and $(n - 1)$ positive eigenvalues such that for all $x \in R^n$ and $\xi \in (-\infty, +\infty)$

$$2x^* H\,[(A + \lambda I)\,x + b\xi] - c^* x \xi \leqslant -\delta\,(c^* x)^2 \tag{4.2}$$

It follows from inequality (4.2) that $2Hb = c$. Hence

$$\det\left(H - \frac{cc^*}{2c^* b}\right) = \det H \det\left(I - \frac{H^{-1} cc^*}{2c^* b}\right) = \det H \left(1 - \frac{c^* H^{-1} c}{2c^* b}\right) = 0$$

This relationship and the fact that H has one and only one negative eigenvalue and $\det H \neq 0$ yields the inequality $H - (2c^* b)^{-1} cc^* \geqslant 0$. From which it immediately follows that Condition (3) of Theorem 2 is satisfied, if $W(t) = x(t)^* H x(t)$, $v(\sigma) \equiv -(2c^* b)^{-1}$, and $x(t)$ is some solution of system (4.1).

Let us now assume that $f(\sigma) = \varphi(\sigma)$, $\psi(t) = \varphi(\sigma(t)) - \delta \sigma^{\cdot}(t) + g(t)$, and $\sigma(t)$ is some solution of system (4.1). Then from inequality (4.2) we obtain

$$W^{\cdot}(t) + 2\lambda W(t) \leqslant \psi(t)\, \sigma^{\cdot}(t), \qquad \forall t \geqslant 0$$

from which it follows that for $u(\sigma) \equiv \lambda$ Condition (5) of Theorem 2 is satisfied. The presence of function $F(\sigma)$ in Conditions (1) and (2) of Theorem 2 follows from Condition (4) of Theorem 4 [7].

Hence, if the inequalities

$$c^* x(0) > 0, \quad x(0)^* H x(0) < -{}^1\!/_2\, F(\sigma(0))^2 \tag{4.3}$$

are satisfied for solution $x(t)$ and $\sigma(t)$, that solution is unbounded in the interval $(0, +\infty)$. It is clear from the conditions for the spectrum of matrix H that there exists vector $x(0)$ and number $\sigma(0)$ that satisfy inequalities (4.3).

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